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A dark-bright optical soliton solution to the coupled nonlinear Schrödinger equation

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Abstract

A sufficiently general form of a two-component dark-bright (DB) vector optical soliton with seven free real parameters is obtained by solving the integrable coupled nonlinear Schrödinger equation (Manakov model) with the help of the Hirota method. We find that this solution unlike the other vector soliton solutions of the Manakov model (namely bright–bright, dark–dark and bright-guided dark vector optical solitons) possesses a singularity. This singularity is found to have a restricted movement in the parametric domain of the solution provided the dark component of the DB vector soliton is a gray dark soliton. However in the case of the DB soliton with fundamental dark component, this singularity is fixed to a particular value. In addition, two different physically interesting cases namely DB soliton with self-focusing nonlinearity and self-defocusing nonlinearity arise under two different parametric conditions. Finally, its collision dynamics is also investigated by constructing a more general DB multisoliton solution.

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The study of coupled nonlinear Schrödinger (CNLS) equations has received a great deal of attention in recent years due to their appearance as modeling equations in diverse areas including nonlinear optics [1–9]. The incoherently CNLS equations used to describe the (1+1)-dimensional propagation of high-intensity pico-second light of arbitrary polarization in the isotropic Kerr media [10] reduce to the well-known integrable vector soliton system [11] (namely Manakov model)

$$\mathrm{i}\frac{\partial u_m}{\partial z} + \frac{\partial^2 u_m}{\partial x^2} - 2\mu(|u_m|^2 + |u_{3-m}|^2)u_m = 0, \qquad m = 1, 2, \tag{1}$$

if the ratio between nonlinearity due to self-phase modulation (SPM) (the coefficient of $|u_m|^2$) and cross-phase modulation (CPM) (the coefficient of $|u_{3-m}|^2$) is unity. In real materials the

SPM/CPM ratio can take on a wide range of values. The case SPM/CPM =1 is a special one possessing integrability property. Therefore, considerable attention was initially paid to derive its different kinds of exact vector soliton (shape-preserving coupled soliton) solutions of (1) mathematically [11–14]. Later on in a few physical situations, like photorefractive crystals [2] and AlGaAs crystal at frequencies near half the band gap [1], the spatial vector solitons of (1) have been experimentally observed. Here, u_1 and u_2 are the two components of vector soliton; z and x are the longitudinal and transverse coordinates; the nonlinear coefficient $\mu < 0$ [12] (or $\mu > 0$ [13]) if the Manakov model (1) governs the spatial vector solitons in the self-focusing (SF) (or self-defocusing (SDF)) Kerr media. Without loss of generality, one can remove μ from (1) by properly scaling the components u_1 and u_2. However, we retain μ here to conveniently define the existing regions of the reported vector soliton solution.

The exact vector soliton solutions of (1) with enough degrees of freedom have richer propagation dynamics than their scalar NLS equations (one-component counterparts) because of their multicomponent nature. For example, the bright–bright (BB) vector soliton solution of (1) [11] (which is valid only when $\mu < 0$), differs from its scalar counterparts by exhibiting the energy exchange collision in the SF Kerr media [12]. This cascaded collision dynamics of BB spatial vector soliton lays experimental foundation for optical computation [7] and information transfer [8]. Moreover, it was investigated in diverse areas [3–6, 9] including Bose–Einstein condensates [5]. In addition to this BB vector soliton in the SF Kerr media ($\mu < 0$), the bright guided dark (BGD) (i.e., bright component does not exist without a dark component's support) [13] and the eight-parameter dark–dark (DD) [15] vector soliton solutions in the SDF Kerr media ($\mu > 0$) were also realized by solving (1) with $\mu > 0$. However to the best of our knowledge so far a more general spatial vector optical soliton solution in the SDF Kerr media with the energy exchange collision property and the dark–bright (DB) vector optical soliton with enough degrees of freedom were not realized by solving the integrable CNLS family of equations. Can one possibly find such solutions by using (1)? In order to answer this question, we have performed the following investigations.

In this paper, we are able to realize a new class of two-component vector optical soliton solution in the Manakov model (1) through the Hirota method. It has nontrivial coupling between dark and bright components (i.e., between the dark and the bright soliton solutions of the scalar NLS equations, respectively, with the SDF nonlinearity and the SF nonlinearity) having same envelope width, envelope speed and envelope trough location. Further, unlike the BGD vector soliton solution of (1) [13], each of its component exists in the absence of other, but with different nonlinear effects. As the solution couples two such component solitons with SF and SDF nonlinearities, it has a singularity in between two parametric domains with $\mu > 0$ and $\mu < 0$. These two parametric domains actually correspond to two different special cases of the solution (namely seven-parameter DB vector soliton with the SDF nonlinearity and the SF nonlinearity) realized under two different parametric conditions. One can satisfy the parametric condition by the suitable choice of seven free parameters of the DB vector soliton solution. In addition, we have classified our solution into DB soliton with fundamental dark pulse and gray dark pulse based on its minimum intensity value (i.e., minimum intensity of fundamental dark soliton is zero while it is nonzero minimum in the gray dark soliton case). While in the former case the singularity is fixed to a particular value, it can be moved towards the SDF domain by tuning the free parameters in the later. Finally, by constructing a more general DB vector multisoliton solution of (1) we have noted both the beating and breathing effects as in the BGD case [13]. However, in our case, one can eliminate such effects without disturbing the envelope width and speed of each colliding DB vector 1-soliton, which was not possible in the BGD case. Further the present work enables us to realize the energy exchange collision not only in the SF media but also in the SDF media by using the many component vector soliton system obtained by generalizing (1). However, it is beyond the scope of this paper to report this, due to the complicated mathematics involved with many components. Here, we have focused our attention in introducing a new class of two-component DB vector soliton solution with the above interesting properties by solving (1). Here, Hirota's technique as shown below is used for this purpose.

By using the Hirota bilinear transformations [14],

$$u_1 = \frac{g}{f} \qquad \text{and} \qquad u_2 = \frac{h}{f}, \tag{2}$$

the Manakov equation (1) can be decoupled as the following set of bilinear equations:

$$\begin{aligned} &\left(\mathrm{i}D_z + D_x^2 - \lambda\right) g \cdot f = 0, \qquad \left(\mathrm{i}D_z + D_x^2 - \lambda\right) h \cdot f = 0 \qquad \text{and} \\ &\left(D_x^2 - \lambda\right) f \cdot f = -2\mu(gg^* + hh^*), \end{aligned} \tag{3}$$

where $*$ denotes the complex conjugate, λ is an unknown decoupling constant, $g(z, x)$ and $h(z, x)$ are complex functions while $f(z, x)$ is a real function and the Hirota bilinear operators D_z and D_x are defined by $D_z^m D_x^n (g \cdot f) = (\partial_z - \partial_{z'})^m (\partial_x - \partial_{x'})^n [g(z, x) \cdot f(z', x')]|_{z=z', x=x'}$.

The above set of equations can be solved by introducing the following power series expansions for g, h and f:

$$g = g_0(1 + \chi^2 g_2), \qquad h = \chi h_1 \qquad \text{and} \qquad f = 1 + \chi^2 f_2, \tag{4}$$

where χ is the formal expansion parameter. The resulting set of equations, after collecting the terms with the same power in χ, can be solved recursively to obtain g_0, g_2, h_1 and f_2. There are many ways to define g_0, g_2, h_1 and f_2. But the Hirota method needs judicious ansatz for input functions g_0 and h_1 to provide practically interesting nontrivial coupled solution [13, 14] or solution with more number of arbitrary parameters [12, 15]. In this paper, we are able to realize a new class of coupled DB vector optical soliton by following the Hirota method [14] with $g_0 = \tau(R)^{-1/2}\exp(\mathrm{i}\psi)$ and $h_1 = \alpha_1 \exp(\eta_1)$, where $\tau = \tau_R + \mathrm{i}\tau_I$ and $\alpha_1 = \alpha_{1R} + \mathrm{i}\alpha_{1I}$ are complex parameters, $\psi = lx - (l^2 + \lambda)z + \psi^{(0)}$ and $\eta_1 = k_1 x + \mathrm{i}\left(k_1^2 - \lambda\right)z + \eta_1^{(0)}$ in which l and $\psi^{(0)}$ are real parameters, $k_1 = k_{1R} + \mathrm{i}k_{1I}$ and $\eta_1^{(0)} = \eta_{1R}^{(0)} + \mathrm{i}\eta_{1I}^{(0)}$ are complex parameters and R and λ are the parameters to be determined. The resultant solution has nontrivial coupling between the dark and bright components having the same envelope width (k_{1R}), the envelope speed (k_{1I}) and the envelope trough location ($\eta_{1R} + \Gamma/2 = 0$) with seven free parameters as

$$\begin{aligned} u_1 &= \cos\theta\, \mathrm{e}^{-\mathrm{i}\delta z}\, \mathrm{e}^{2\mathrm{i}A^2 z} A[\mathrm{i}\sin\beta + \cos\beta\tanh(\eta_{1R} + \Gamma/2)]\, \mathrm{e}^{\mathrm{i}(\psi\prime + \beta + \phi_1)}, \\ u_2 &= \sin\theta\, \mathrm{e}^{-\mathrm{i}\delta z}\, \mathrm{e}^{2\mathrm{i}A^2 z} A \sec h(\eta_{1R} + \Gamma/2)\, \mathrm{e}^{\mathrm{i}(\eta\prime_{1I} + \phi_2)}, \end{aligned} \tag{5}$$

where $\theta = \arctan(|\alpha_1|/2|\tau|)$, $\phi_1 = \arctan(\tau_I/\tau_R)$, $\phi_2 = \arctan(\alpha_{1I}/\alpha_{1R})$, $\psi' = lx - l^2 z + \psi^{(0)}$, $\beta = \arctan[(k_{1I} - l)/k_{1R}]$, $\eta'_{1I} = k_{1I}x + \left(k_{1R}^2 - k_{1I}^2\right)z + \eta_{1I}^{(0)}$, $\eta_{1R} = k_{1R}(x - 2k_{1I})z + \eta_{1R}^{(0)}$, $\Gamma = \ln(R)$, $\delta = \left\{\left(2|\alpha_1|^2 k_{1R}^2\right)\big/[(4|\tau|^2 + |\alpha_1|^2)\Delta']\right\}$ and $A = [k_{1R}(\mu\Delta')^{-1/2}]$. Here $R = \left\{[\mu\Delta'(4|\tau|^2 + |\alpha_1|^2)]\big/4k_{1R}^2\right\}$, $\Delta' = (\cos^2\theta\cos^2\beta - \sin^2\theta)$ and $\lambda = 2A^2 - \delta$.

There are seven free real parameters namely $k_{1R}, k_{1I}, l, \tau_R, \tau_I, \alpha_{1R}$ and α_{1I} in the above solution. It can be characterized as explained below. From the envelope phase ($\eta_{1R} + \Gamma/2$) of (5), one can easily recognize k_{1R} and k_{1I} as the two independent parameters for the envelope width and envelope speed. Therefore, it is possible to change the envelope width without affecting the envelope speed of (5) or vice versa. Next to understand the role of l, the changes in u_1 of (5) while β varies as a function of k_{1R}, k_{1I} and l should be examined. It is interesting to note that with respect to the value of β the vector soliton (5) can be classified into (i) DB vector soliton with fundamental dark component under the condition $k_{1I} = l$ (i.e., at $\beta = 0$), (ii) DB vector soliton with gray dark component under the condition $k_{1I} \neq l$ (i.e., $\beta \neq 0$). That is, if $\beta = 0$, u_1 component reduces its form without disturbing u_2 as

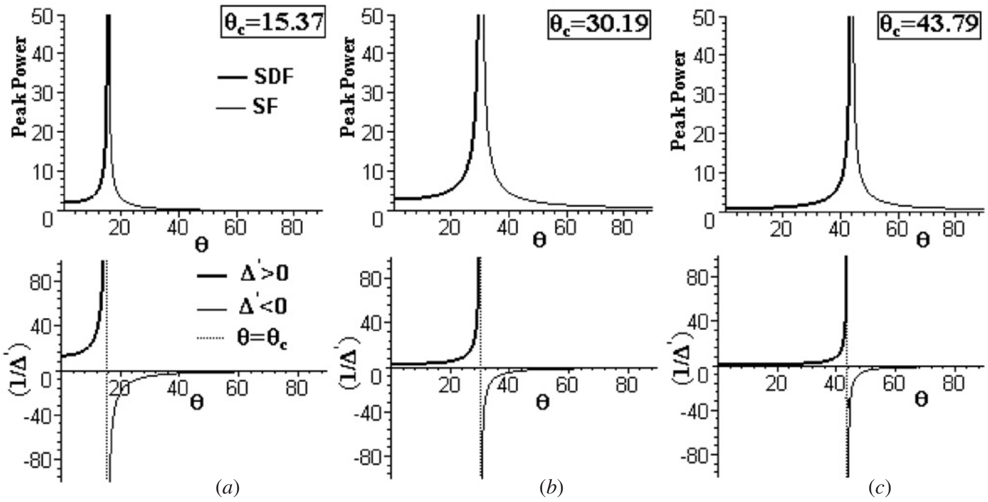


Figure 1. The peak power variation against θ in degree for $l = 0.5$, $\mu = 1.0$, (a) $k_{1R} = 0.2$ and $k_{1I} = 1.2$; (b) $k_{1R} = 0.5$ and $k_{1I} = 1.2$; (c) $k_{1R} = 1.0$ and $k_{1I} = 0.8$.

$u_1 = \cos\theta\, \mathrm{e}^{-\mathrm{i}\delta z}\, \mathrm{e}^{2\mathrm{i}A^2 z} A \tanh(\eta_{1R} + \Gamma/2)]\, \mathrm{e}^{\mathrm{i}(\psi' + \phi_1)}$ and which is called as the fundamental dark soliton since its minimum intensity is zero at $\eta_{1R} + \Gamma/2 = 0$. In the $\beta \neq 0$ case, the minimum intensity of the dark component of (5) is nonzero. However it can be varied by tuning β with the help of k_{1R}, k_{1I} and l. If the value of β is changed by using the free parameter l, then the minimum intensity of u_1 component of (5) varies without affecting the envelope width and the envelope speed. The remaining four free parameters are coupling parameters and which can be characterized as explained below.

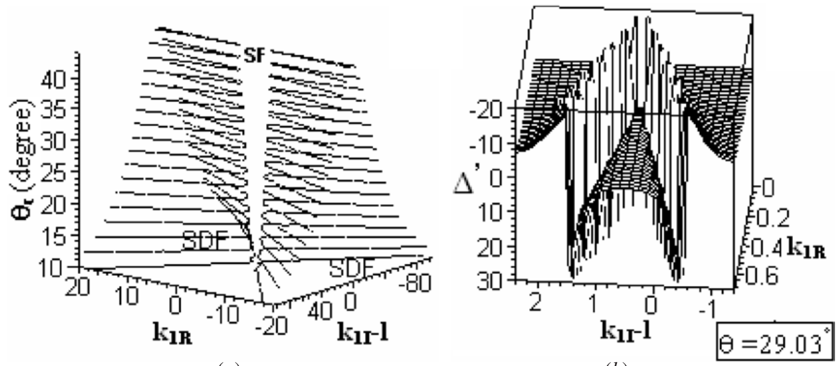
It is interesting to note from (5) that the DB vector soliton is valid with the above free parameters if $\mu\Delta' > 0$ (i.e., signs of μ and Δ' must be same). This condition is needed here in order to satisfy Γ with positive real R. Consequently with respect to the value of Δ', the following cases can be realized from (5) as

- DB soliton with the SDF nonlinearity ($\mu > 0$) under the condition $\Delta' > 0$ (i.e., $\cos^2\theta \cos^2\beta > \sin^2\theta$),
- DB soliton with the SF nonlinearity ($\mu < 0$) if $\Delta' < 0$ (i.e., Δ' is negative if $\cos^2\theta\cos^2\beta < \sin^2\theta$),
- The singularity of DB soliton at $\cos^2\theta\cos^2\beta = \sin^2\theta$.

The singularity of the solution can be written as

$$\theta = \theta_{\mathrm{c}} = \arctan\left[\frac{k_{1R}^2}{k_{1R}^2 + (k_{1I} - l)^2}\right]^{1/2}, \tag{6}$$

where θ_{c} is a critical value of θ at which the DB soliton solution (5) becomes extinct. It is obvious to note from (6) that in the case of DB soliton with fundamental dark soliton (i.e., at $k_{1I} = l$) the value of θ_{c} is fixed to a particular value 45°. Otherwise (if $k_{1I} \neq l$) the value of θ_{c} is decided by the choice of k_{1R}, k_{1I} and l. In the general solution (5) the value of θ is selected by using α_1 and τ. Hence, if we choose α_1 and τ such that $\theta > \theta_{\mathrm{c}}$ or $\theta < \theta_{\mathrm{c}}$ then (5) occurs with the SF nonlinearity ($\Delta' < 0$) or with the SDF nonlinearity ($\Delta' > 0$). This is shown in figures 1(a)–(c) by plotting the peak power of DB soliton (i.e., the maximum value of total intensity $|u|^2 = |u_1|^2 + |u_2|^2$) and the value of $1/\Delta'$ against θ. From these figures, we note that



(a) (b)

Figure 2. The contour plot obtained by plotting (a) equation (6) and (b) Δ' in (5). (Note that $\Delta' > 0$ represents SDF and $\Delta' < 0$ defines SF regions).

the singularity of (5) appears in between the existing regions of (5) and moves with respect to the values of k_{1R}, k_{1I} and l as for as $k_{1I} \neq l$. However, such a movement occurs with in the limit $\theta < \theta_c < 45°$ due to the nature of (6). That is in (6) the term $\left[k_{1R}^2 + (k_{1I} - l)^2\right]^{-1/2}$ is greater than k_{1R} as for as $k_{1I} \neq l$. Therefore, from (6) it is obvious that θ_c moves only in between 0° and 45° with respect to the values of k_{1R}, k_{1I} and l as for as $k_{1I} \neq l$. Such movement can be characterized by plotting (6) as shown in figure 2.

Figure 2(a) is a contour plot and has a hyperbola in its θ_c plane. It represents a trajectory in between the existing regions of (5). By plotting Δ' for any given θ (as shown in figure 2(b)) one can identify that the area within the width of hyperbola has $\Delta' > 0$ (SDF nonlinearity) and the region in between the vertices of hyperbola has $\Delta' < 0$ (SF nonlinearity). From the figure 2, one can also note that the distance between the vertices of the hyperbola decreases as θ_c tends to zero. At $\theta_c = 0°$, the existing region appears only within the width of hyperbola supporting the SDF nonlinearity. It confirms a fact that when $\theta = 0°$ the solution (5) decouples and has only the scalar dark soliton in u_1 with the SDF nonlinearity. Further, one can note that when $\theta = 90°$, the solution (5) decouples and supports only bright scalar soliton in u_2 with the SF nonlinearity. When these two scalar solitons with different Kerr nonlinearities are coupled, then only the term δ starts to appear in the complex modulation in z. From the expression for δ one can also note that the term δ has Δ'. Therefore, the complex modulation in z varies with respect to the sign of μ. Moreover, the term A in the amplitude part of coupled soliton (5) depends on the all free parameters and also has a singularity. Therefore, the total peak power of (5) depends on the all seven free parameters.

The total intensity of DB soliton can be defined as $|u|^2 = |u_1|^2 + |u_2|^2$ and by using (5) the relation for $|u|^2$ can be easily derived. We note that, such a relation depends on θ as much as the dependence of A on θ. Therefore, the variation in the peak power or maximum value of $|u|^2$ against θ is unavoidable if A includes θ. Suppose we were to remove θ from A by using a constraint $\mu\Delta' = 1$, then we can distribute the peak power $\left(k_{1R}^2 = A^2\right)$ of $|u|^2$ between the u_1 and u_2 components as $k_{1R}^2 \cos^2\theta$ and $k_{1R}^2 \sin^2\theta$, respectively. This is possible as the peak power of the DB soliton is conserved against θ under the restriction $\mu\Delta' = 1$. This is numerically confirmed as shown in figure 3 by plotting the peak power of (5) against θ under the constraint $\mu\Delta' = 1$.

The collisions between many DB 1-solitons can be analyzed by constructing a DB multisoliton solution. For this purpose by following the systematic steps of Hirota method

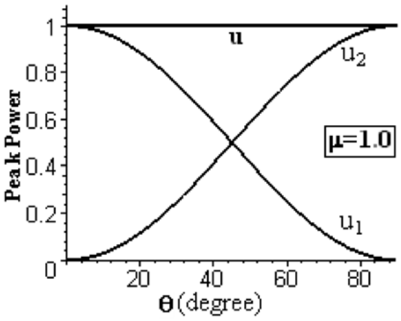


Figure 3. Note that the peak power which is conserved against θ under a constraint is distributed among u_1 and u_2 by tuning θ. Here we take $k_{1R} = 1.0$, $k_{1I} = -1.0$ and $\mu = 1.0$.

[14], we define the expression g, h and f (in $u_1 = g/f$ and $u_2 = h/f$) correspond to the N-soliton solution as

$$g = \frac{\tau}{\sqrt{R}} e^{i\psi} \sum M_1(a) \exp\left[\sum_{j=1}^{2N} a_j(\eta_j + \zeta_j) + \sum_{1\leqslant j<k}^{2N} a_j a_k A_{jk}\right]$$
$$h = \sum M_2(a) \exp\left[\sum_{j=1}^{2N} a_j(\eta_j + \varepsilon_j) + \sum_{1\leqslant j<k}^{2N} a_j a_k A_{jk}\right] \qquad (7)$$
$$f = \sum M_1(a) \exp\left[\sum_{j=1}^{2N} a_j\eta_j + \sum_{1\leqslant j<k}^{2N} a_j a_k A_{jk}\right].$$

Here, the first sum is over all possible permutations of the vector $a = (a_1, a_2, \ldots, a_{2N})$ in which a_j's can be either 0 or 1. Then

$$M_1 = \begin{cases} 1 & \text{if} \quad \sum_{j=1}^{N} a_j = \sum_{j=1+N}^{2N} a_j, \\ 0 & \text{otherwise,} \end{cases}$$
$$M_2 = \begin{cases} 1 & \text{if} \quad \sum_{j=1}^{N} a_j = \sum_{j=1+N}^{2N} a_j + 1, \\ 0 & \text{otherwise.} \end{cases}$$

The other parameters are defined as $\eta_j = \eta_j, \eta_{j+N} = \eta_j^*, e^{\varepsilon_j} = \alpha_j, e^{\varepsilon_{j+N}} = \frac{1}{\alpha_j^*}, e^{\zeta_j} = cg_j, e^{\zeta_{j+N}} = -\frac{1}{cg_j^*}, e^{A_{jk}} = \upsilon_{jk}, e^{A_{j+N,k+N}} = \upsilon_{jk}^*, e^{A_{jk+N}} = R_{jk}$ and $e^{A_{j+Nk}} = R_{jk}^*$ in which $\eta_j = k_j x + i(k_j^2 - \lambda)z + \eta_j^{(o)}, \psi = lx - (l^2 + \lambda)z + \psi^{(o)}, cg_j = k_{jR} + i(k_{jI} - l), \upsilon_{jk} = \frac{-(k_j-k_k)^2}{\alpha_j\alpha_k}\left[\frac{|\tau|^2}{cg_j cg_k^* R} + \frac{1}{\mu}\right], R_{jk} = \left[\frac{\mu|\tau|^2}{cg_j cg_k^* R/R_{jk}} - \frac{\mu\alpha_j\alpha_k^*}{(k_j+k_k^*)^2}\right]$ and $R = \prod_{j,k=1}^{N} R_{jk}$. Here the suffixes R and I represent the real and imaginary parts, respectively, and $*$ represents the complex conjugation. The value of N defines the number of one solitons in the solution (7). It has free parameters k_j, α_j, l and τ where $j = 1, 2, \ldots, N$. Here l and τ are common for all colliding solitons. Because, the Hirota method restricts us to define the same background field for the dark components of all colliding DB 1-solitons. Therefore, each colliding soliton of (7) has a form of (5) in which free parameters are k_j, α_j, τ and l, $j = 1, 2, \ldots, N$. The above solution exhibits both the beating and breathing effects as explained below.

We have noted that the beating effect appears only if the colliding solitons of equation (7) move slowly with approximately equal pulse widths. But the breathing effect appears between

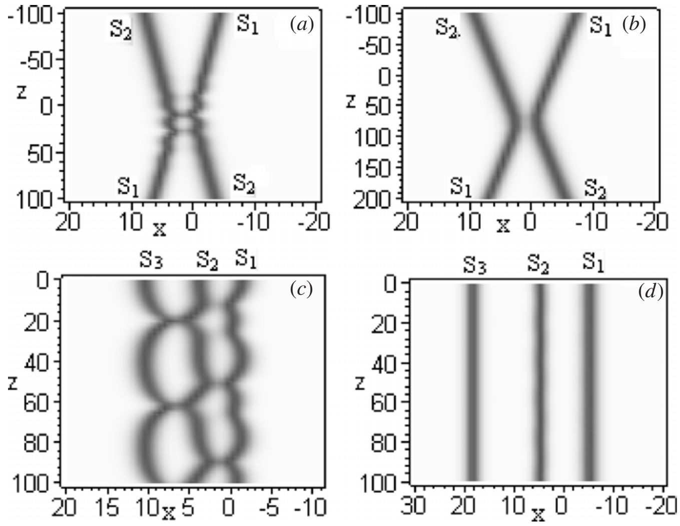


Figure 4. The collision dynamics of equation (7) with the beating and breathing effects. (*a*) Beating effect when $\tau = 1.0, k_{1R} = 0.9, k_{1I} = 0.02, k_{2R} = 0.7, k_{2I} = -0.02, l = 0.2, \alpha_1 = 0.7 + 0.72\mathrm{i}, \alpha_2 = 0.44 + 0.9\mathrm{i}$ and $N = 2$. (*b*) Beating effect is eliminated by tuning α_2 to the value $0.1 + 0.2\mathrm{i}$. (*c*) Breathing effect with $\tau = 1.0, k_{1R} = 0.9, k_{2R} = 0.8, k_{3R} = 0.7, k_{1I} = k_{2I} = k_{3I} = 0.0, l = 0.2, \alpha_1 = 1.0, \alpha_2 = 1.0, \alpha_3 = 1.0$ and $N = 3$. (*b*) Breathing effect is nullified by defining $\alpha_1 = 4.0$ and $\alpha_2 = 0.1$.

stationary DB solitons (i.e., $k_{jI} = 0, j = 1, 2, \ldots, N$) or between the DB solitons moving with equal speed (i.e., $k_{1I} = k_{2I} = \cdots = k_{NI}$) provided they have small pulse width difference. However such effects can be eliminated by tuning the coupling parameters α_j and τ. For instance, the beating and breathing effects and its eliminations are shown in figure 4 by plotting $|u_1|^2$ of equation (7).

In the conclusion, to the best of our knowledge an exact vector soliton whose regions of existence depend on its initial parametric values is realized for the first time with seven free parameters by solving equation (1). It differs from the other well-known vector solitons of equation (1) by admitting a movable singular point in between the existing parametric regions of equation (5) and may receive considerable attention in a variety of nonlinear Kerr media [1–9], where u_1 and u_2 are incoherently coupled as shown by equation (1). It is to be mentioned that we are able to realize the energy exchange collision by extending the present work to the general N-component integrable case suitably. It may lay a strong mathematical foundation in the development of the optical computation [7], matter-wave switching devices with Bose–Einstein condensates [5], etc. The details of such study will be focused elsewhere.

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